



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

TECHNICAL MATHEMATICS P2

NOVEMBER 2022

MARKS: 150

TIME: 3 hours

This question paper consists of 18 pages and a 2-page information sheet.

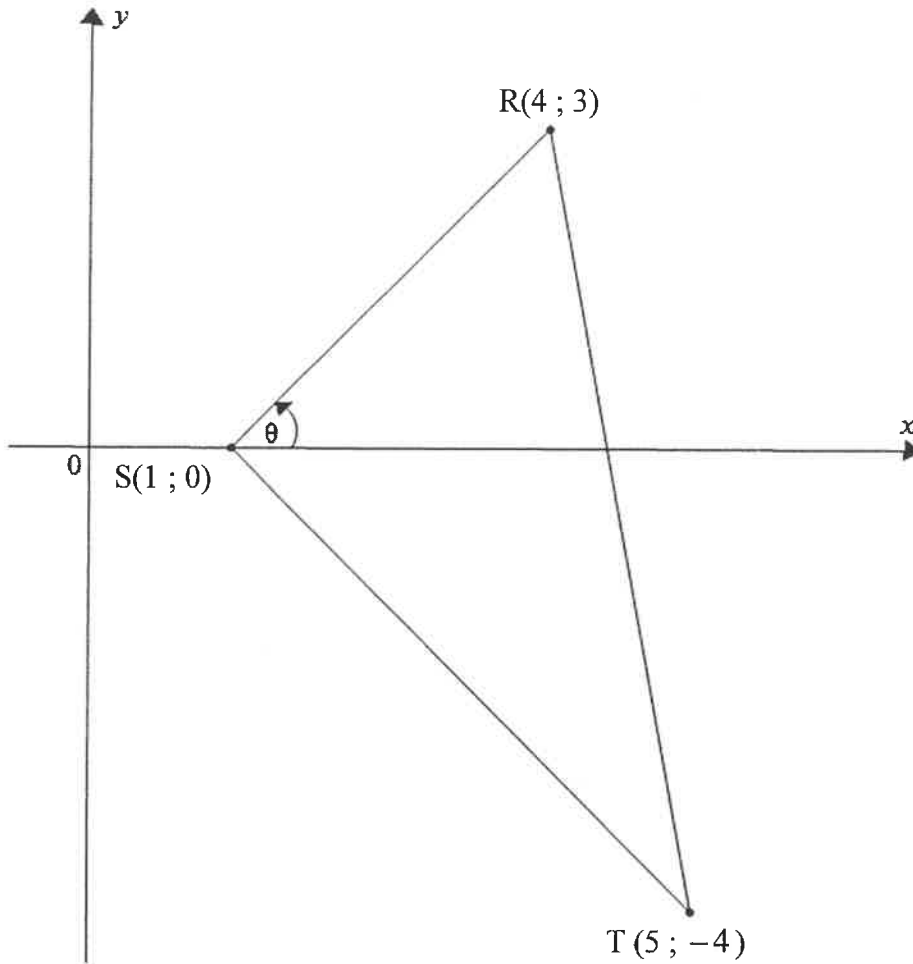
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

In the diagram below, ΔRST with vertices $R(4 ; 3)$, $S(1 ; 0)$ and $T(5 ; -4)$ is given. The angle of inclination of RS with the positive x -axis is θ .



- 1.1 Determine the gradient of RS . (2)
- 1.2 1.2.1 Write down the formula required to calculate the angle of inclination of a line. (1)
- 1.2.2 Hence, determine the value of θ . (2)
- 1.3 Calculate the length of RT in simplified surd form. (2)
- 1.4 Determine the coordinates of the midpoint of ST . (2)

1.5 A line is drawn parallel to RS passing through the midpoint of ST.

1.5.1 Complete the statement:

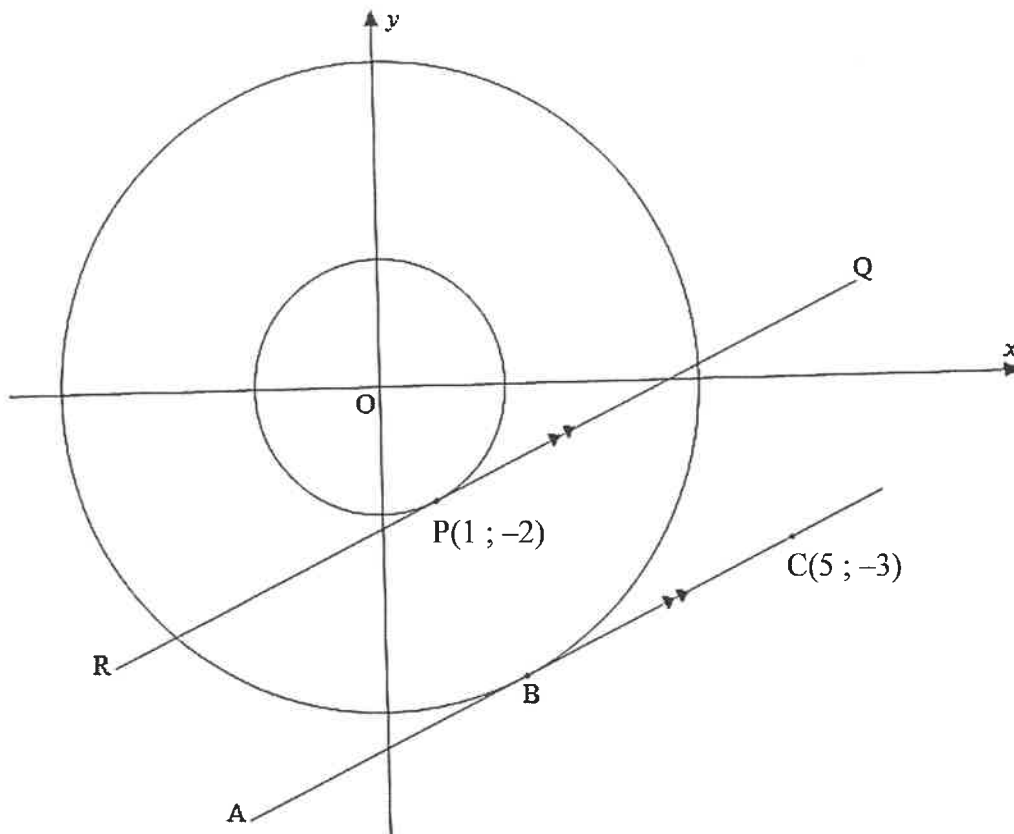
If two lines are parallel, then their gradients are ... (1)

1.5.2 Hence, determine the equation of the line parallel to RS passing through the midpoint of ST in the form $y = \dots$ (3)

[13]

QUESTION 2

- 2.1 In the diagram below, O is the centre of both the smaller and the larger circles.
 RQ is a tangent to the smaller circle at point P(1 ; -2).
 AC is a tangent to the larger circle at point B with C(5 ; -3).
 RQ || AC



- 2.1.1 Determine the equation of the smaller circle. (2)
- 2.1.2 Write down the gradient of OP. (1)
- 2.1.3 Give a reason why OP is perpendicular to RQ. (1)
- 2.1.4 Hence, determine the gradient of AC. (2)
- 2.1.5 Hence, determine the equation of AC in the form $y = \dots$ (3)

2.2 Given: $\frac{x^2}{36} + \frac{y^2}{16} = 1$

2.2.1 Express the equation in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (1)

2.2.2 Hence, sketch the graph defined by $\frac{x^2}{36} + \frac{y^2}{16} = 1$ (2)
[12]

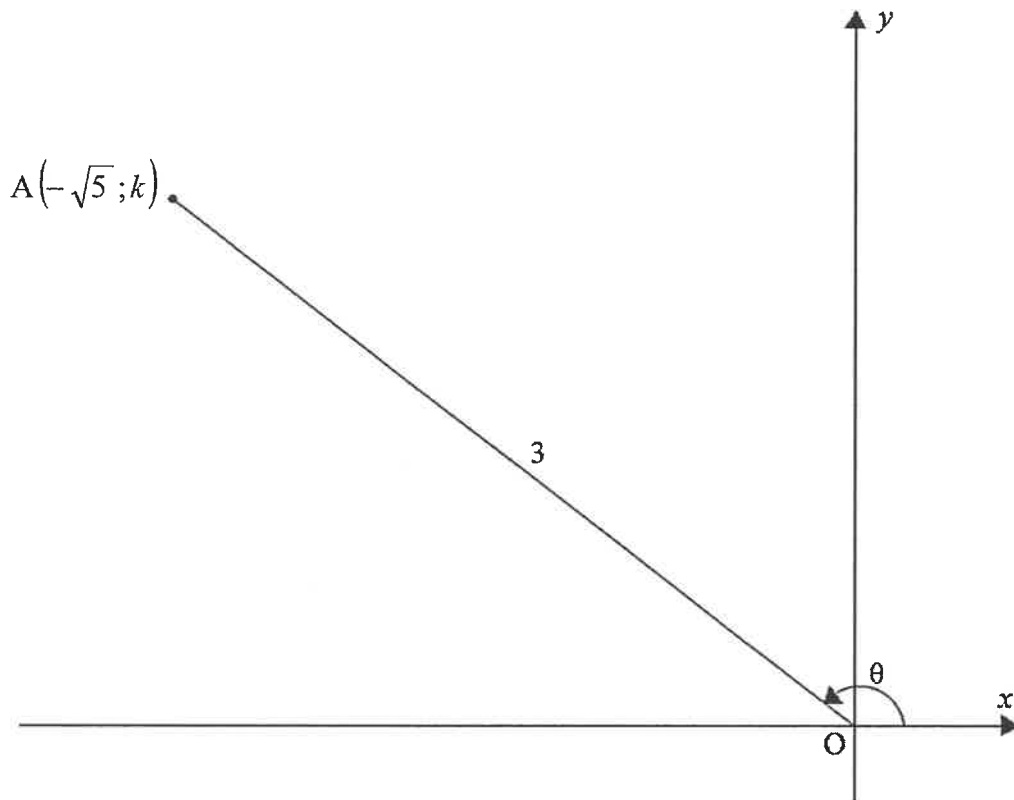
QUESTION 3

3.1 Given: $\hat{P} = \frac{2}{7}\pi$ and $\hat{Q} = 37^\circ$

3.1.1 Convert $\frac{2}{7}\pi$ to degrees. (1)

3.1.2 Determine the value of $\operatorname{cosec} P - \cos Q$. (2)

3.2 In the diagram below, $OA = 3$ units and $A(-\sqrt{5}; k)$ is a point on the Cartesian plane. The angle of inclination of OA with respect to the positive x -axis is θ .



Determine, **without the use of a calculator**, the numerical value of:

3.2.1 k (3)

3.2.2 $\sqrt{5} \cot \theta + 1$ (3)

3.3 Determine the value(s) of x if $3 \tan x = -0,531$ and $x \in [0^\circ; 360^\circ]$. (4)
[13]

QUESTION 4

4.1 Simplify the following:

4.1.1 $\sin(360^\circ - \alpha)$ (1)

4.1.2 $\tan^2(\pi - \alpha)$ (2)

4.1.3 $\frac{\sin(360^\circ - \alpha) \cdot \tan(180^\circ - \alpha) \cdot \operatorname{cosec}(2\pi - \alpha)}{\cos(360^\circ + \alpha) \cdot \operatorname{cosec}(180^\circ - \alpha) \cdot \tan^2(\pi - \alpha)}$ (6)

4.2 Complete the identity: $1 - \sin^2 x = \dots$ (1)

4.3 Prove the identity that: $\operatorname{cosec} x - \sin x = \cot x \cdot \cos x$ (4)
[14]

QUESTION 5

Given the functions defined by $f(x) = \sin(x + 30^\circ)$ and $g(x) = \cos x$ for $x \in [0^\circ; 360^\circ]$

5.1 Write down:

5.1.1 The period of g (1)

5.1.2 The amplitude of f (1)

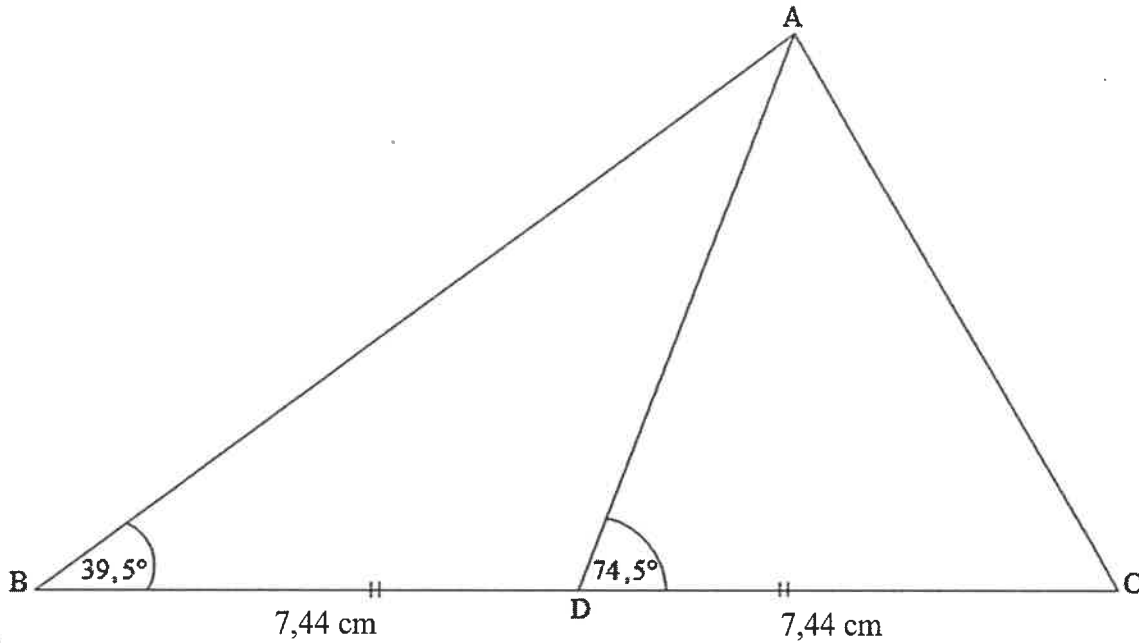
5.2 Draw a sketch graph of f and g on the same set of axes on the grid provided in the ANSWER BOOK. Clearly indicate ALL turning points, end points and intercepts with the axes. (7)

5.3 Use the graph in QUESTION 5.2 to write down the value(s) of x for which the graph of g is increasing. (2)
[11]

QUESTION 6

In the diagram below, BDC is a straight line with $BD = DC = 7,44$ cm

$\hat{B} = 39,5^\circ$ and $\hat{ADC} = 74,5^\circ$



6.1 Determine the size of:

6.1.1 \hat{BAD} (1)

6.1.2 \hat{ADB} (1)

6.2 Consider $\triangle ABD$.

6.2.1 Complete the following with respect to $\triangle ABD$: $\frac{\dots}{\sin \hat{BDA}} = \frac{BD}{\dots}$ (1)

6.2.2 Hence, determine the length of AB. (2)

6.3 Hence, calculate the length of AC. (3)

6.4 Determine the area of $\triangle ABC$. (3)

[11]

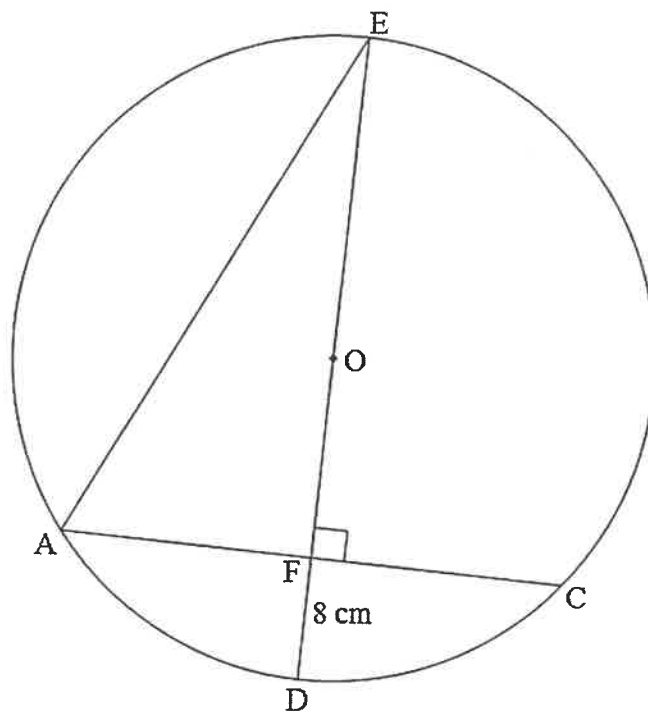
Give reasons for your statements in QUESTIONS 7, 8 and 9.

QUESTION 7

7.1 Fill in the missing word(s) in the following theorem statement:

The ... of a chord passes through the centre of the circle. (1)

7.2 In the diagram below, AC and AE are chords of the circle with centre O.
Diameter ED is perpendicular to AC at F.
ED = 34 cm, FD = 8 cm and AC = 30 cm



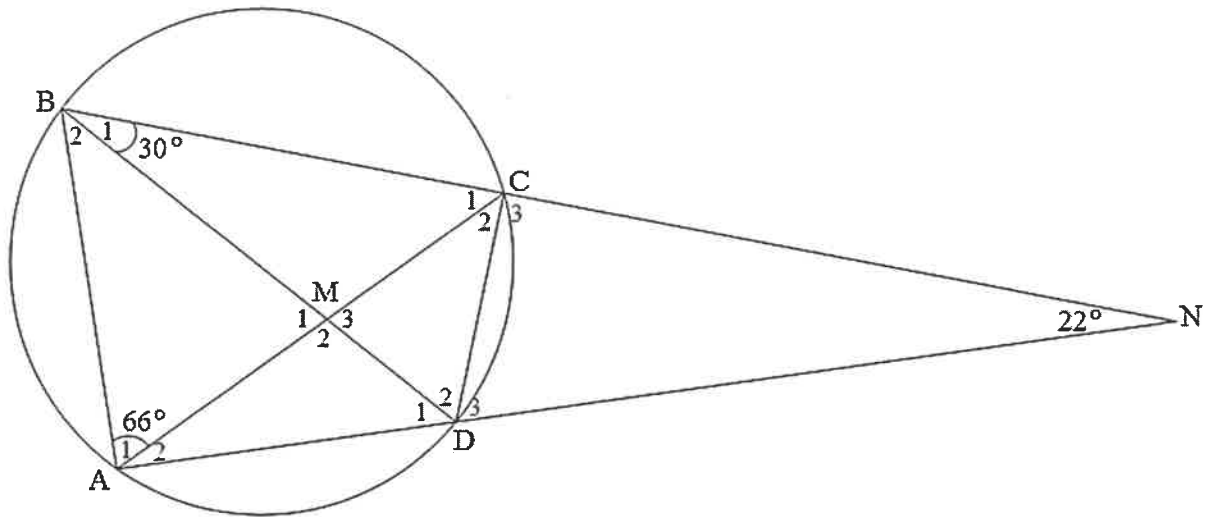
Determine, stating reasons, the length of AE. (5)
[6]

QUESTION 8

8.1 Complete the following theorem statement:

The exterior angle of a cyclic quadrilateral is equal to the ... (1)

8.2 In the diagram below, ABCD is a cyclic quadrilateral. Diagonals BD and AC intersect at M. Chords BC and AD produced meet at N such that $\hat{N} = 22^\circ$, $\hat{B}_1 = 30^\circ$ and $\hat{A}_1 = 66^\circ$.



8.2.1 Determine, stating reasons, the size of the following angles:

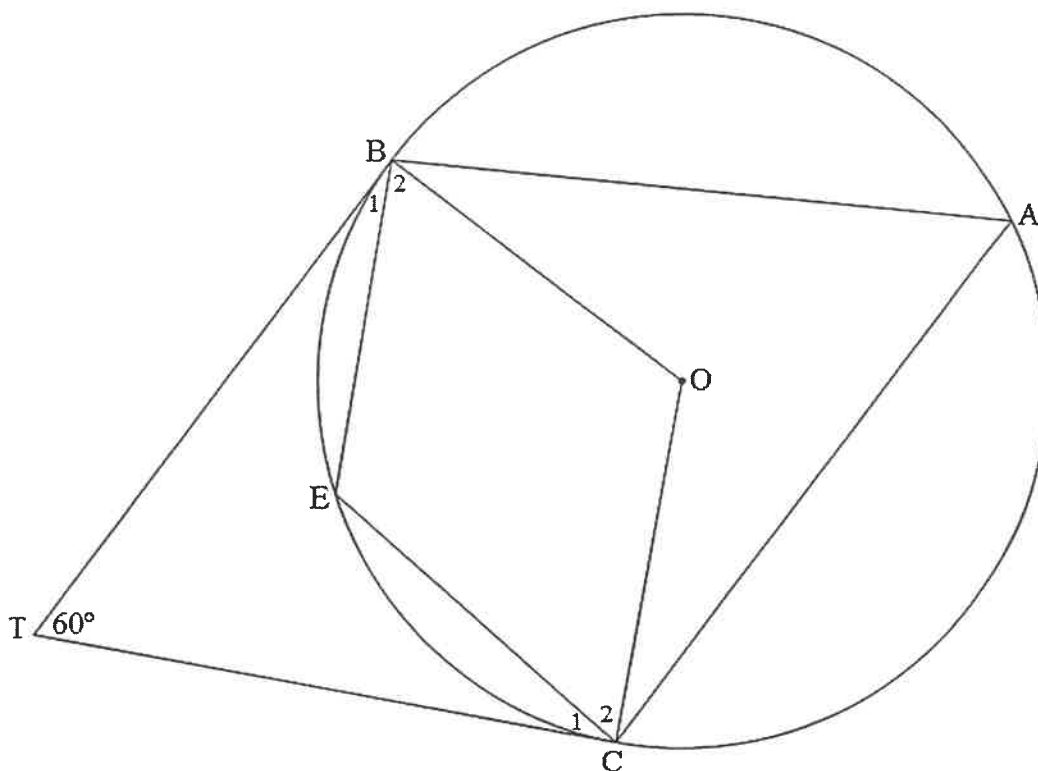
(a) \hat{A}_2 (2)

(b) \hat{C}_1 (2)

(c) \hat{C}_3 (2)

8.2.2 Show, stating reasons, that quadrilateral MCND is NOT a cyclic quadrilateral. (2)

- 8.3 In the diagram below, O is the centre of circle ABEC.
 Tangents TB and TC touch the circle at B and C respectively, such that $\hat{T} = 60^\circ$.
 Radii OB and OC are drawn.



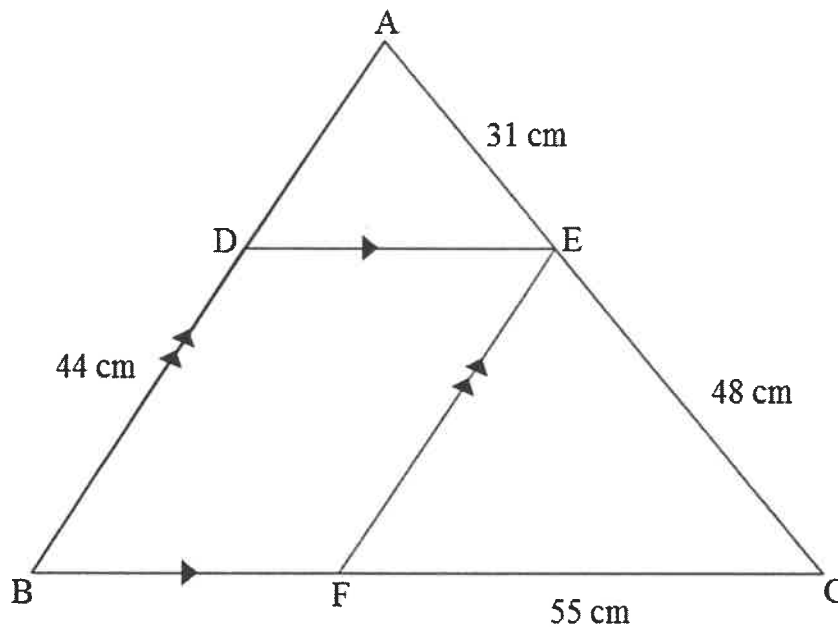
- 8.3.1 Write down, stating reasons, TWO angles each equal to 90° . (3)
- 8.3.2 Determine, stating reasons, the size of the following angles:
- (a) \hat{A} (4)
- (b) \hat{E} (2)
- [18]**

QUESTION 9

9.1 Complete the following theorem statement:

A line drawn parallel to one side of a triangle divides the other two sides ... (1)

9.2 The diagram below shows $\triangle ABC$ with points D, E and F on sides AB, AC and BC respectively. DEFB is a parallelogram.
 AE = 31 cm, EC = 48 cm, BD = 44 cm and FC = 55 cm.

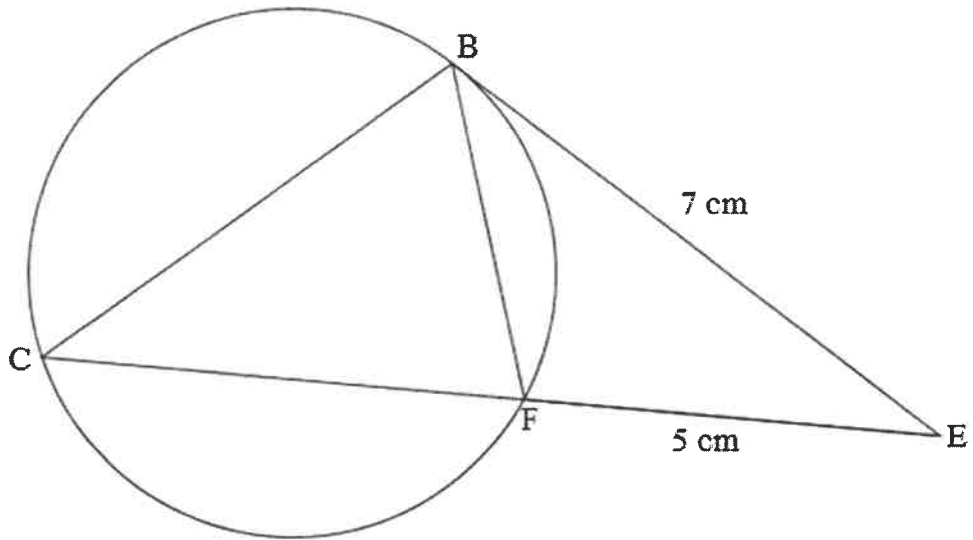


9.2.1 If $DE \parallel BF$ and $BD \parallel FE$, state any TWO OTHER properties of the parallelogram. (2)

9.2.2 Determine, stating reasons, the length of AD. (3)

9.2.3 Determine the length of DE. (3)

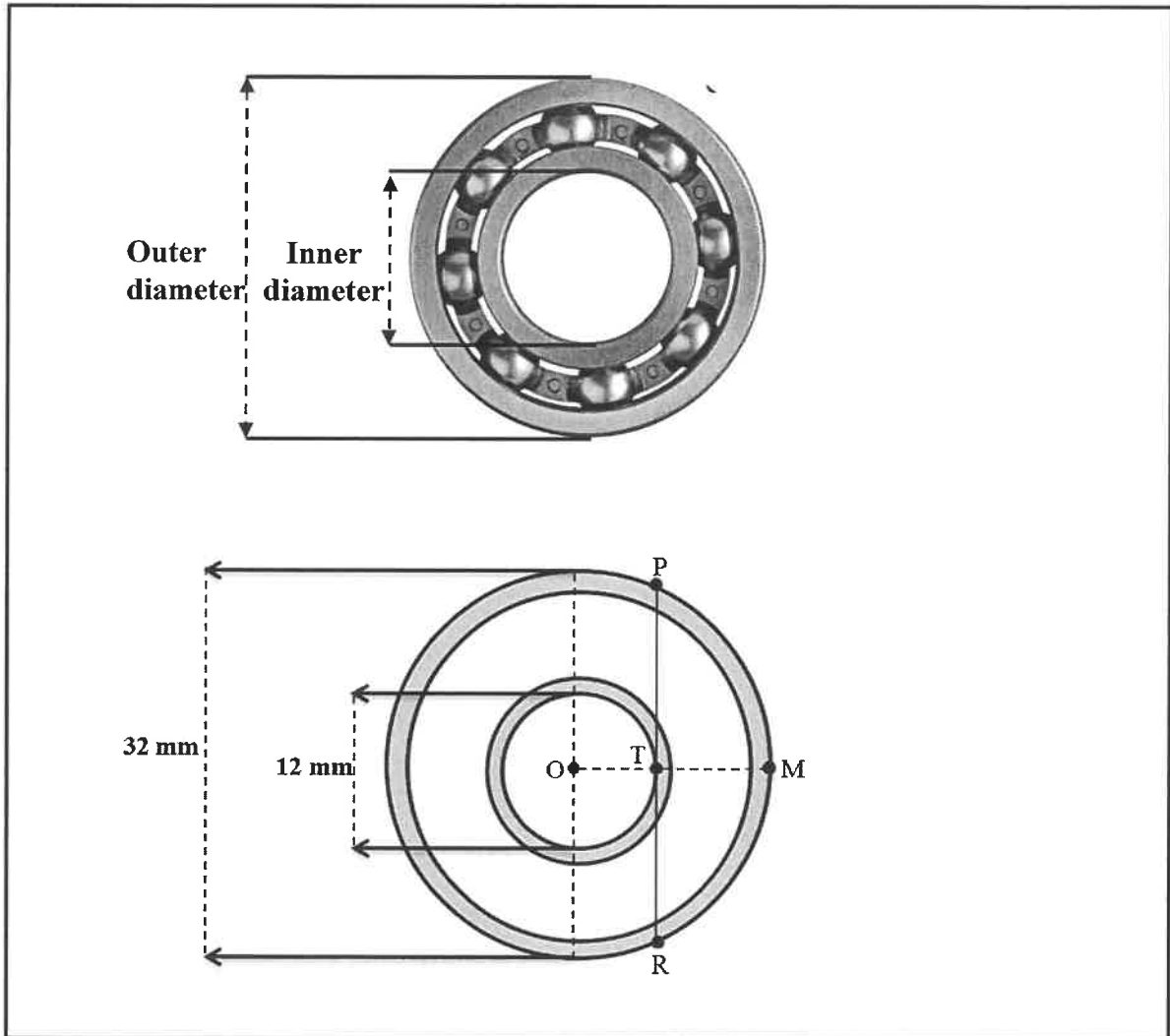
- 9.3 In the diagram below, BE is a tangent to circle BCF at point B.
Chord CF produced meets BE at E such that BE = 7 cm and FE = 5 cm.



- 9.3.1 Prove that $\Delta EBF \sim \Delta ECB$. (4)
- 9.3.2 Hence, deduce that $EB^2 = EC \times EF$ (1)
- 9.3.3 Determine the length of CF. (3)
- [17]**

QUESTION 10

- 10.1 Below is a picture of a bearing. The diagram below the picture models the bearing with an outer diameter of 32 mm and an inner diameter of 12 mm.
 O is the centre of the circles.
 P, M and R are points on the circumference of the bigger circle.
 PR is a chord of the outer circle and is a tangent to the inner circle at point T.
 TM is the height of segment PMR.

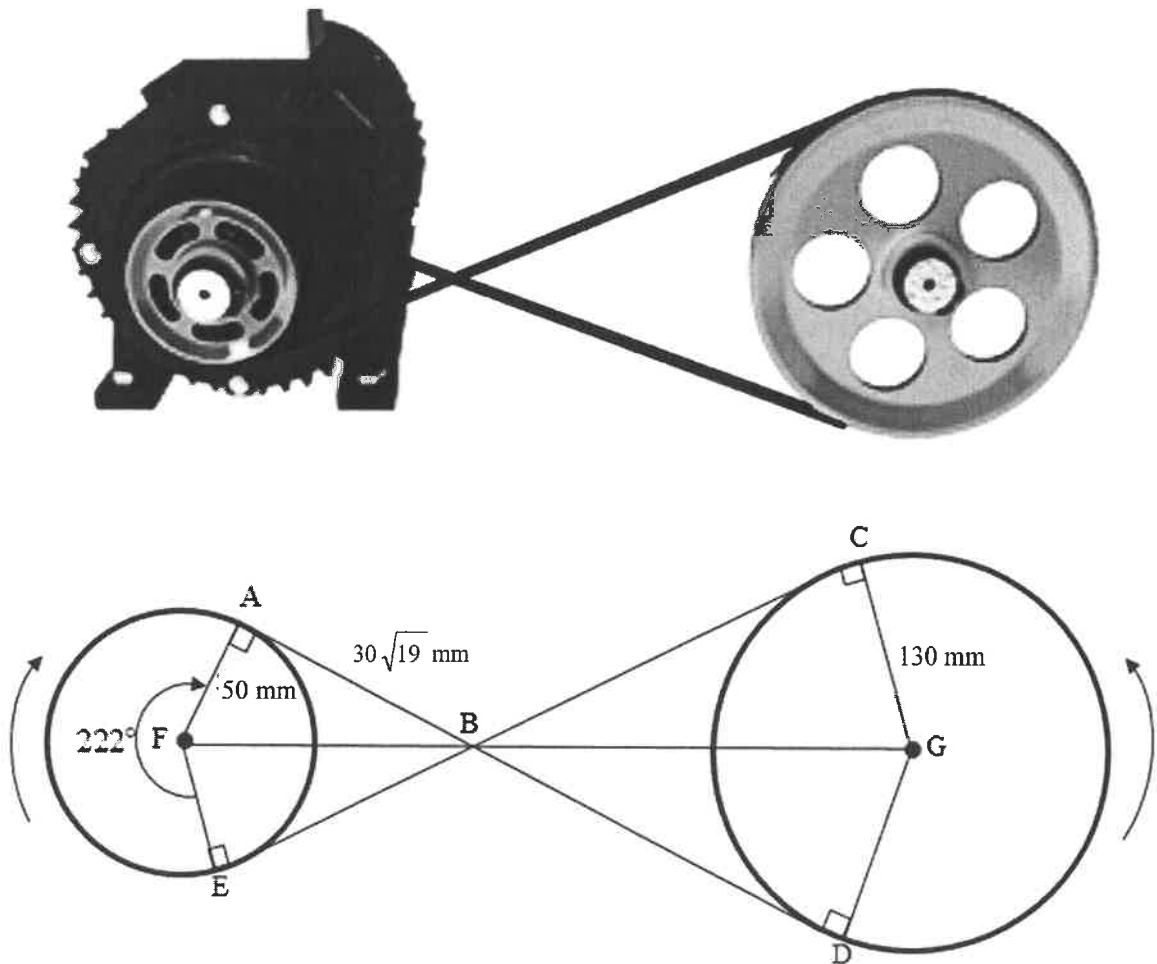


Determine:

- 10.1.1 The circumferential velocity (in metres per second) of a particle at point M on the bearing when it rotates at 5 000 revolutions per minute (4)
- 10.1.2 The length, in millimetres, of chord PR (5)

- 10.2
- The picture and the diagram below show a belt crossing at point B around two pulleys with centres F and G which are 500 mm apart.
 - The smaller pulley is connected to a motor.
 - As the smaller pulley, with centre F and a radius of 50 mm rotates, it causes the larger pulley, with centre G and a radius of 130 mm, to rotate in the opposite direction.
 - Reflex $\hat{A}FE = 222^\circ$
 - $AB = 30\sqrt{19}$ mm

The diagram shows the two parallel pulleys.

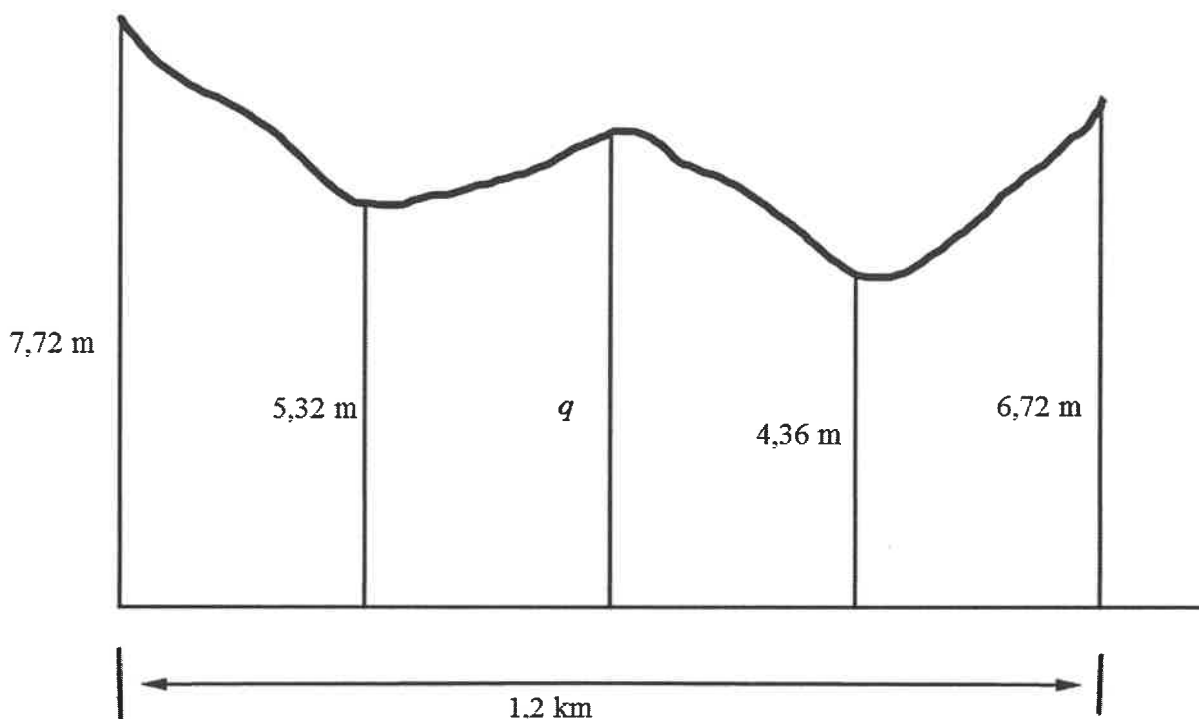


- 10.2.1 Convert 222° to radians. (1)
- 10.2.2 Calculate the area of major sector AE. (3)
- 10.2.3 Determine the total length of the crossed belt if the length of the major arc CD is 503 mm. (7)
- [20]

QUESTION 11

11.1 A farmer has an irregular piece of land on his farm that he wants to use. He determines that one straight side of the land is 1,2 km in length.

He divides this straight side of the land into four equal segments, resulting in five different ordinates of lengths, 7,72 m, 5,32 m, q , 4,36 m and 6,72 m, as shown in the diagram below.

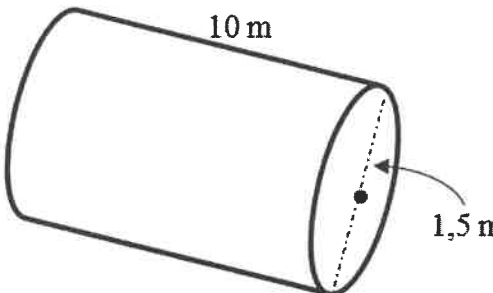



11.1.1 Convert 1,2 km to metres. (1)

11.1.2 If the area of the irregular piece of land is $6\,948\text{ m}^2$, determine the numerical value of q . (5)

11.2 A fuel station uses a horizontal right cylindrical tank to store fuel underground. The storage tank is filled to contain at most 68 m^3 of fuel (FIGURE A). The fuel tank of a car is filled to contain at most 52 litres of fuel (FIGURE B).

- The height of the cylindrical storage tank is 10 m and the radius is 1,5 m.
- The capacity of the right cylindrical storage tank is $70,69 \text{ m}^3$
- The capacity of the car fuel tank is 55 litres.

FIGURE A	FIGURE B
	
<div style="border: 1px solid black; padding: 5px; display: inline-block;"> $V = 70,69 \text{ m}^3$ </div>	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> $V = 55 \text{ litres}$ </div>
<p>Total surface area of a closed right cylinder = $2\pi r^2 + 2\pi r h$</p> <p>Volume of a right cylinder = $\pi r^2 h$</p>	

11.2.1 The right cylindrical storage tank is covered with special material to prevent leakage. The material used costs R8,93 per square metre.

Show that the cost of material used to cover the right cylindrical tank will not exceed R1 000. (4)

11.2.2 The right cylindrical storage tank and the car fuel tank have air space when filled to the given capacity.

Determine which ONE of the tanks will have a bigger percentage air space. (5)

[15]

TOTAL: 150

INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a}$$

$$y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 + i)^n$$

$$A = P(1 - i)^n$$

$$i_{eff} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\int kx^n dx = \frac{kx^{n+1}}{n+1} + C, \quad n, k \in \mathbb{R} \text{ with } n \neq -1 \text{ and } k \neq 0$$

$$\int \frac{k}{x} dx = k \ln x + C, \quad x > 0 \text{ and } k \in \mathbb{R}; k \neq 0$$

$$\int k a^{nx} dx = \frac{k a^{nx}}{n \ln a} + C, \quad a > 0; a \neq 1 \text{ and } k, a \in \mathbb{R}; k \neq 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\tan \theta = m$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area of } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$\pi \text{ rad} = 180^\circ$$

$$\text{Angular velocity} = \omega = 2 \pi n \quad \text{where } n = \text{rotation frequency}$$

$$\text{Angular velocity} = \omega = 360^\circ n \quad \text{where } n = \text{rotation frequency}$$

$$\text{Circumferential velocity} = v = \pi D n \quad \text{where } D = \text{diameter and } n = \text{rotation frequency}$$

$$\text{Circumferential velocity} = v = \omega r \quad \text{where } \omega = \text{angular velocity and } r = \text{radius}$$

$$\text{Arc length} = s = r\theta \quad \text{where } r = \text{radius and } \theta = \text{central angle in radians}$$

$$\text{Area of a sector} = \frac{rs}{2} \quad \text{where } r = \text{radius, } s = \text{arc length}$$

$$\text{Area of a sector} = \frac{r^2 \theta}{2} \quad \text{where } r = \text{radius and } \theta = \text{central angle in radians}$$

$$4h^2 - 4dh + x^2 = 0 \quad \text{where } h = \text{height of segment, } d = \text{diameter of circle} \\ \text{and } x = \text{length of chord}$$

$$A_T = a(m_1 + m_2 + m_3 + \dots + m_n) \quad \text{where } a = \text{number of equal parts, } m_1 = \frac{o_1 + o_2}{2} \\ O_n = n^{\text{th}} \text{ ordinate and } n = \text{number of ordinates}$$

OR

$$A_T = a \left(\frac{o_1 + o_n}{2} + o_2 + o_3 + \dots + o_{n-1} \right) \quad \text{where } a = \text{number of equal parts, } o_n = n^{\text{th}} \text{ ordinate} \\ \text{and } n = \text{number of ordinates}$$