



NATIONAL SENIOR CERTIFICATE EXAMINATION
SUPPLEMENTARY EXAMINATION – MARCH 2019

MATHEMATICS: PAPER I

Time: 3 hours

150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 11 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. Answer all the questions.
4. Number your answers exactly as the questions are numbered.
5. You may use an approved non-programmable and non-graphical calculator unless otherwise stated.
6. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.

Answers only will NOT necessarily be awarded full marks.

7. Diagrams are not necessarily drawn to scale.
 8. If necessary, round off answers to **ONE** decimal place, unless stated otherwise.
 9. It is in your own interest to write legibly and to present your work neatly.
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SECTION A**QUESTION 1**

- (a) Consider the following arithmetic sequence:
 $(x+5)$; $(37-x)$; $(x+13)$...
- (1) Determine the value of x . (3)
- (2) Determine the general term of the sequence in the form: $T_n = \dots$ (2)
- (b) The sum of the first three terms of a geometric sequence is 91 and its common ratio is 3, determine the first term of the sequence. (3)
- (c) In a convergent geometric series, $S_2 = 90$ and $S_\infty = \frac{375}{4}$. Determine its first term and its common ratio. (6)
- (d) The share price of a certain company formed a quadratic pattern over a specific time interval.

The share price at the end of each day for the first 5 days was:

Day 1: R 32 699
Day 2: R 32 896
Day 3: R 33 091
Day 4: R 33 284
Day 5: R 33 475

- (1) Show that the pattern is quadratic. (2)
- (2) Determine a formula for the n^{th} term of the pattern. (6)
- (3) At the end of which day, will the share price be at its maximum? (3)

[25]

QUESTION 2

(a) Given: $f(x) = -x^2 + 2x$. Determine $f'(x)$ from first principles. (5)

(b) If $g(x) = 2x^3 + 3x^2 + 1$. Determine the equation of the tangent to the curve at $x = -2$. (5)

(c) Determine $\frac{dy}{dx}$ for the following:

(1) $y = \sqrt[3]{x^2} + 3x^2 - 4x$ (3)

(2) $y = (x + \pi)^{-1} (x^{-1} + \pi^{-1})$ (4)

[17]

QUESTION 3

Given: $f(x) = 3x^3 + 3$

(a) Show that f is increasing for all values of x . (2)

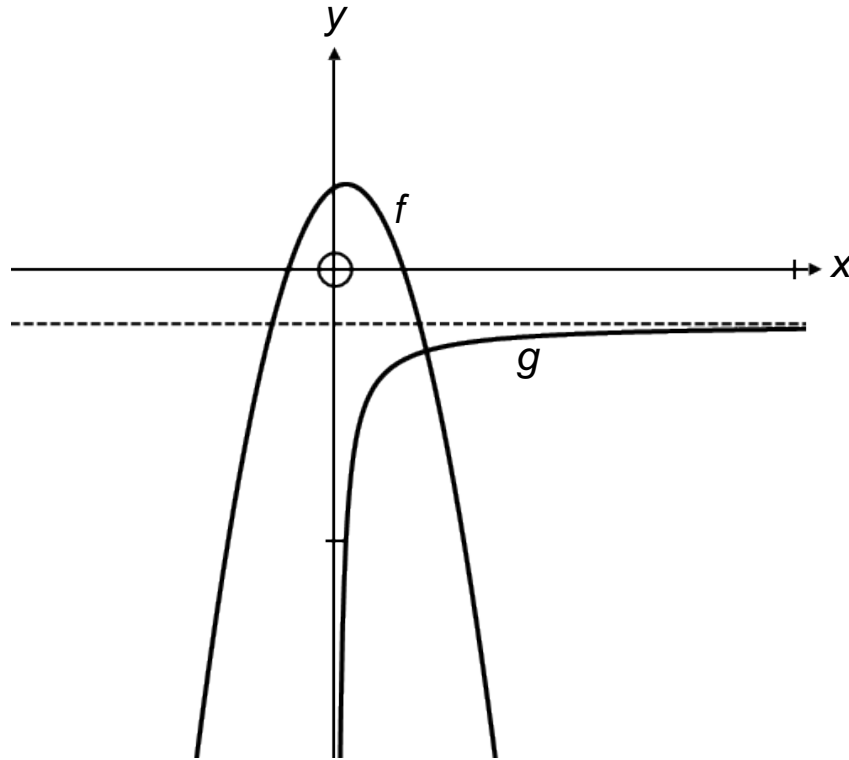
(b) Sketch the graph of $f(x) = 3x^3 + 3$ showing the intercepts with the axes. (4)

(c) Write down and classify the stationary point. (2)

[8]

QUESTION 4

The graphs of a quadratic function $f(x) = ax^2 + bx + c$ and $g(x) = \frac{d}{x} + q$ for $x > 0$, are sketched below.



$f(-1) = 0$, $f(0) = 3$ and the point $(1; 2)$ lies on the graph of f .

- (a) Determine the values of a , b and c . (5)
- (b) Determine the values of x for which the gradient of $f(x)$ is decreasing. (2)
- (c) If the graphs of f and g intersect at the point $(x; -3)$ and the graph of g has a horizontal asymptote at $y = -2$, determine the value of d . (6)

[13]

QUESTION 5

A rabbit population grew exponentially over a period of time. This exponential growth was modelled as follows:

$f(x) = y = 558(1,08)^x$, where x represents the number of years and y the total population.

(a) Determine the approximate number of years it would take for the population to double. (3)

(b) Determine the equation of the inverse of this function in the form: $f^{-1}(x) = \dots$ (2)

[5]

QUESTION 6

The following set of numbers is given: $\{2;4;6;9;10;14;15;16;18\}$

If subset A represents all the even numbers and subset B represents all the square numbers:

(a) Represent the given information by way of a Venn Diagram. (4)

(b) Determine $P(A \text{ or } B)$ (1)

(c) Determine $P(A \text{ and } B)$ (1)

(d) Determine $P(A' \text{ and } B')$ (1)

[7]

75 marks

SECTION B**QUESTION 7**

- (a) Three pupils' mathematics results were analysed over a period of time. Based on the analysis, it was predicted that the probability that each will pass the next examination is $\frac{4}{5}$, $\frac{5}{7}$ and $\frac{2}{3}$ respectively.

Based on the predicted probabilities,

- (1) determine the probability that all three will pass the examination. (2)
- (2) determine the probability that all three do not pass the examination. (3)
- (3) determine the probability that at least one will pass the examination. (1)

- (b) Mr Botha realised that he had forgotten his password to log in to internet banking. His password is made of 5 letters, the first and last letters being the same and the middle letters different to each other and different to the first and last. He has three attempts before being locked out of his account.

All answers should be accurate and not rounded.

- (1) How many passwords can he choose from? (2)
- (2) Determine the probability that he will unlock the account on the first attempt. (1)
- (3) Show mathematically that his chance of being locked out of his account on three attempts is almost 1. (3)

[12]

QUESTION 8

- (a) Greta purchased a car for R325 000, 9 years ago. The car depreciated on a reducing balance at a rate of 7% per annum over the 9 years.

Determine what percentage this depreciation value would be of the car's original value.

(3)

- (b) Greta was granted a loan of R1 825 000 to purchase a house which was repayable over 25 years. The bank offered her an interest rate of 9,5% per annum compounded monthly.

After 9 years of repaying her loan, she decided to purchase a second car.

Instead of applying for vehicle finance she decided to access money from her home loan.

Option 1:

She could access 75% of each of her monthly repayments over the 9 years without interest.

Option 2:

She could access five(5) times the monetary **difference** between the original loan and the balance on the loan after 9 years, immediately after the 108th payment.

Which option would yield the highest amount available to purchase a new car? **Show all working.**

(10)
[13]

QUESTION 9

(a) Solve for x by completing the square, leaving your answers in terms of a .
 $2x^2 - 2x = a$ (4)

(b) Solve for x in terms of p .

(1) $\log_p [x(x+p)] = 0$ (4)

(2) $(x-p+3)^2 = 4$ (3)

(c) Solve for x .

Answers only will not be awarded full marks.

(1) $\sqrt{x+4} = \frac{4}{\sqrt{x-2}}$ (4)

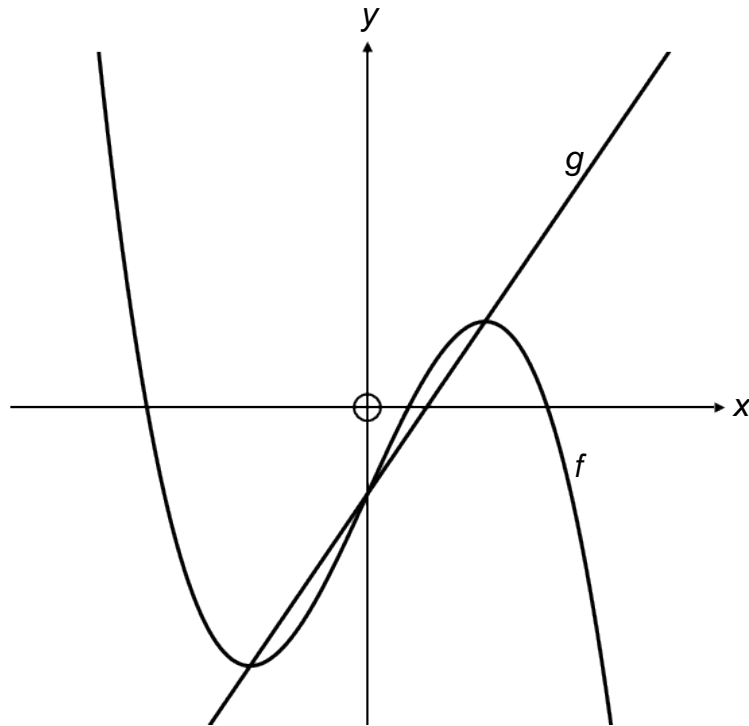
(2) $2^{4x} - 8 \cdot 2^x = 0$ (3)

(3) $3^x (x^2 - 3x + 2) \leq 0$. (4)

(d) Given: $9x^2 - 12px = -4p^2$.
For which value(s) of p will the equation have equal roots? (4)
[26]

QUESTION 10

The graphs of a cubic function f and a straight line g are sketched below.



The equation of the curve is given as $f(x) = -x^3 + bx^2 + cx + d$ and the straight line is given as $g(x) = 2x + d$. f and g intersect at the stationary points of f .

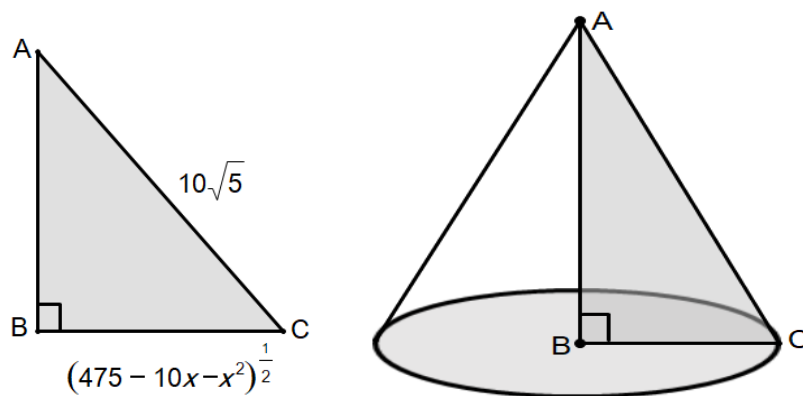
$f'(x) = f'(-x)$ for all real values of x and $f'(-1) = g''(2)$.

- (a) Determine the values of b and c . Show all working. (6)
- (b) Given that $d = -1$,
Sketch the graphs of the inverses of f^{-1} and g^{-1} on the same set of axes. (4)
- [10]**

QUESTION 11

In the diagram, $\triangle ABC$ is a right angled triangle with hypotenuse $10\sqrt{5}$ cm and $BC = (475 - 10x - x^2)^{\frac{1}{2}}$.

The triangle is rotated about BC to form a right circular cone as shown in the diagram.



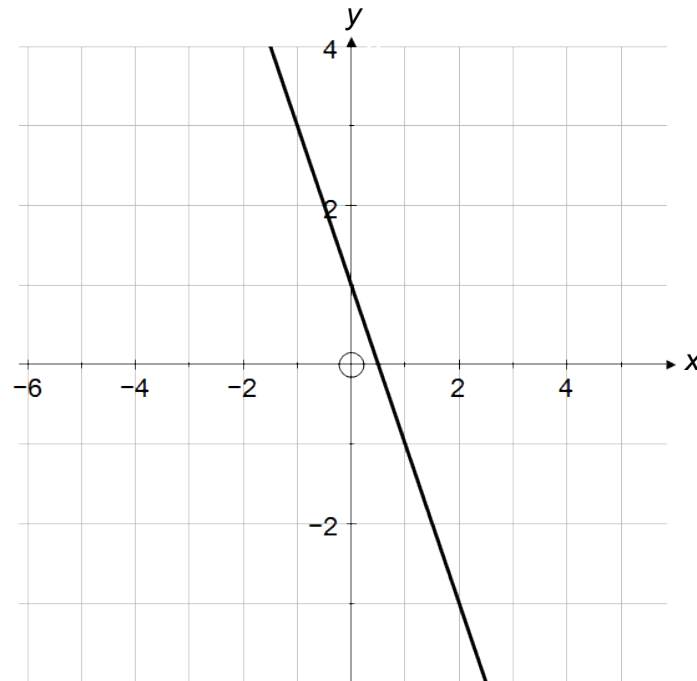
Calculate the maximum volume of the cone.

Useful formula: Volume of a cone $= \frac{1}{3}\pi r^2 h$

[8]

QUESTION 12

The sketch of $f''(x)$, the second derivative of the function f , is given below.
 $f''(x)$ passes through $(0;1)$ and $(1;-1)$.



- (a) Determine the values of x for which the curve f is concave down. (3)
- (b) On the same set of axes draw a sketch graph of f' , the derivative of f . (3)
- [6]**

75 marks

Total: 150 marks