GRADE 12 EXAMINATION
NOVEMBER 2020

ADVANCED PROGRAMME MATHEMATICS: PAPER II

Time: 1 hour | 100 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 17 pages, an Answer Sheet of 1 page (Module 4) and an Information Booklet of 4 pages (i–iv). Please check that your question paper is complete.

2. This question paper consists of THREE modules:

   Choose **ONE** of the **THREE** modules:

   MODULE 2: STATISTICS (100 marks) OR
   MODULE 3: FINANCE AND MODELLING (100 marks) OR
   MODULE 4: MATRICES AND GRAPH THEORY (100 marks)

3. Non-programmable and non-graphical calculators may be used.

4. All necessary calculations must be clearly shown and writing must be legible.

5. Diagrams have not been drawn to scale.

6. **Rounding of final answers.**

   MODULE 2: Four decimal places, unless otherwise stated.

   MODULE 3: Two decimal places, unless otherwise stated.

   MODULE 4: Two decimal places, unless otherwise stated.

MODULE 2 STATISTICS

QUESTION 1

1.1 A collection of antique toys containing six train sets, five model cars and three vintage toys are displayed at an auction. Three of these items are sold. Find the probability that all three items sold are of the same type. (8)

1.2 The results of a random survey show that 70% of students listen to music while studying for an assessment.

(a) A random sample of five students is selected. Find the probability that more than three students listen to music while studying for an assessment. (8)

(b) A second sample of 60 students is selected. The random variable X denotes the number of students who listen to music while studying for an assessment.

(i) Using a suitable approximation, find the probability that at least 40 students of the sample selected listen to music while studying for an assessment. (8)

(ii) Justify the use of the approximation in (i) mathematically. (2)

(iii) Why does this justification in (ii) need to be provided to ensure that the approximation in (i) is suitable? (2)

[28]

QUESTION 2

2.1 The random variable X is normally distributed. The mean is twice the standard deviation. It is given that $P(X > 5{,}2) = 0{,}9$. Find the mean and the standard deviation, correct to two decimal places. (8)

2.2 A six-sided dice is suspected of bias. The dice is thrown 100 times and it is found that it lands on a six 20 times. [Note that a dice is a cube with each side having a different number of dots on it, ranging from one to six].

(a) Calculate a 94% confidence interval for p, the probability of a dice landing on a six on any throw. (6)

(b) Use your answer to (a) to comment on whether the dice may be biased. (2)

[16]

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QUESTION 3

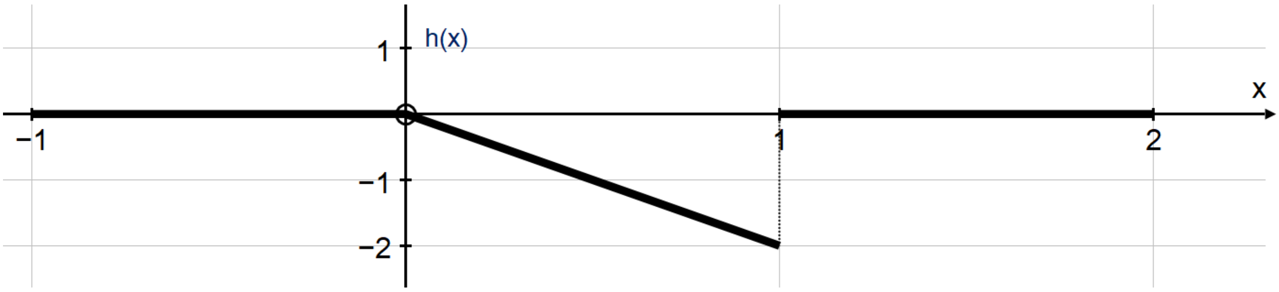
3.1 A player throws three darts. Let X represent the number of darts that hit the bull's eye. The probability distribution of X is shown in the table.

x	0	1	2	3
$P(X = x)$	0,5	0,35	p	q

(a) Given that the expected value of X is 0,67, find the values of p and q. (6)

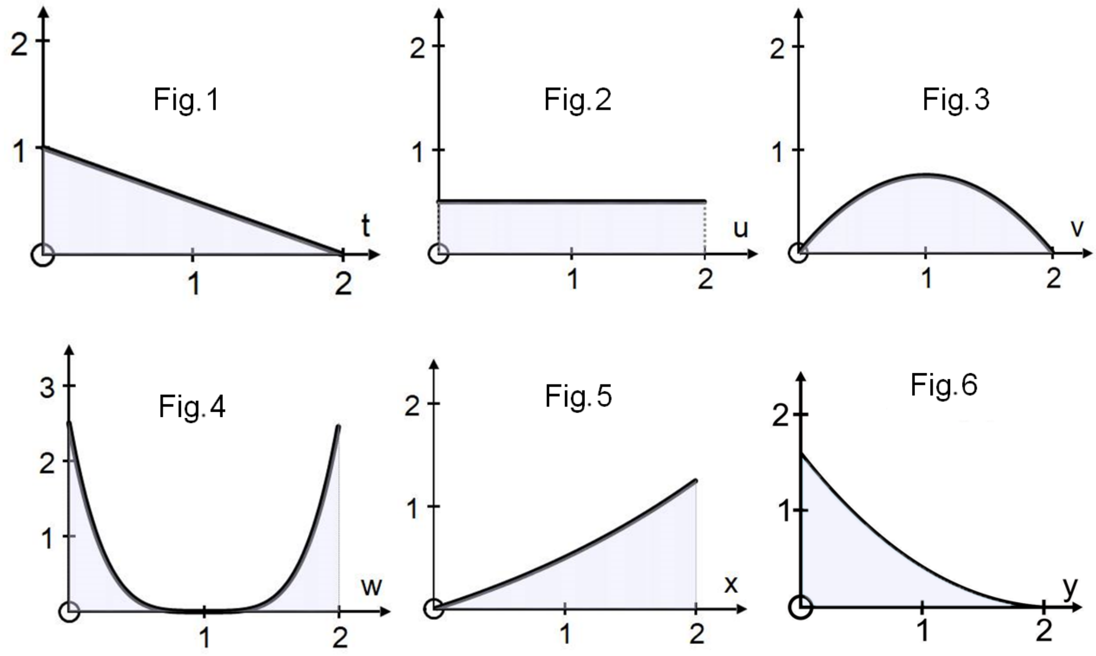
(b) Using the probabilities above and showing all working, find the standard deviation of X. (5)

3.2 The diagrams below show the graphs of two functions, $g(x)$ and $h(x)$. For each of the functions g and h, give a reason why it cannot be a probability density function.



(4)

3.3 Each of the random variables t, u, v, w, x and y takes values between 0 and 2 only. Their probability density functions are shown in Figures 1 to 6 respectively.



(a) Which of these variables has the largest median? Justify your answer. (3)

(b) Which of these variables has the largest standard deviation? Justify your answer. (2)

(c) Use Fig. 1 to find $P(t < 0{,}5)$. (6)

[26]

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QUESTION 4

The manufacturer of a particular mobile phone claims that the mean battery life is 12 hours. A consumer organisation wished to test whether the mean is actually less than 12 hours. They invited a random sample of members to report the battery life of the mobile phones. They then calculated the sample mean. Unfortunately, a fire destroyed the records of this test except for the following partial document.

Test of the mean battery o phone	
Sample size (n)	
Sample mean (*hours*)	11,7
Significant at the 5% level	Yes
Significant at the 2,5% level	No

4.1 State the null and alternate hypotheses that would be used for this test. (2)

4.2 Given that the population of battery lives is normally distributed with a standard deviation of 0,5 hours, find the set of possible values of the sample size, n. (10)

[12]

QUESTION 5

5.1 Consider the game where two people play rock-paper-scissors. Scissors beats paper, paper beats rock and rock beats scissors. Sam and Charlie play a game to decide who gets to wash the dishes after dinner. The loser of the game must wash the dishes.

It is found that Sam chooses rock 36% of the time, paper 32% of the time and scissors 32%. Charlie chooses rock 22% of the time, paper 25% of the time and scissors 53% of the time. If the choices are made independently of each other, what is the probability that Sam has to wash the dishes? [Assume that they only play one round] (8)

5.2 At a final prize-giving ceremony there are three annual sports prizes, one for Best Sportsman, one for Most Improved Sportsman, and one for Best Sportsmanship that are to be awarded in a group of 20 students.

Find the number of different ways in which the three prizes can be awarded if:

(a) no student may win more than one prize. (5)

(b) no student may win all three prizes. (5)

[18]

Total for Module 2: 100 marks

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MODULE 3 FINANCE AND MODELLING

QUESTION 1

Mr Fourie deposits an amount of R1 500 per month in an account, starting immediately, with an effective interest rate of 7,75% per annum. After three years the amount is increased to *x* rand per month with the interest rate unchanged. After a total of seven years, the balance of the account is R206 530,90.

1.1 Determine the balance in the account after three years, correct to the nearest ten rand. (6)

1.2 Assuming that the balance in the account after three years is R62 210, calculate *x*, correct to the nearest rand. (9)

[15]

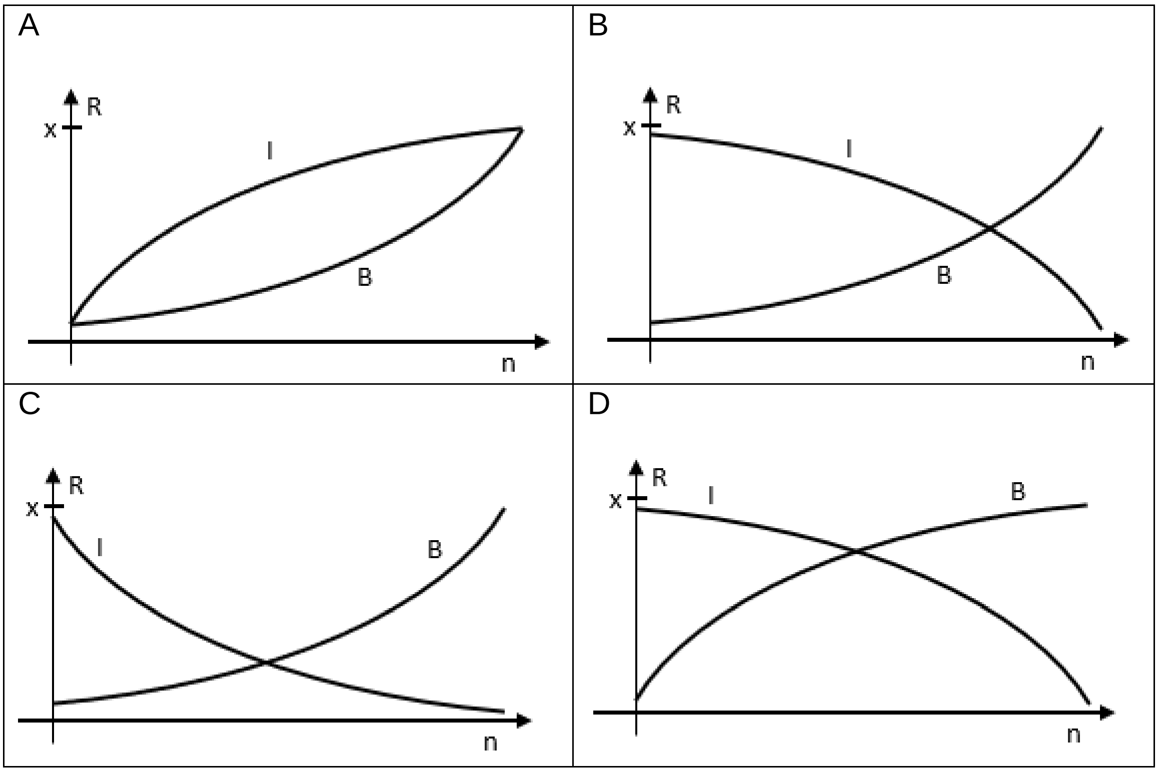
QUESTION 2

2.1 Consider a loan, charging compound interest, that is to be repaid with a monthly payment of *x* rand over *n* months.

Let I = the part of the monthly payment that goes to paying the interest.

Let B = the part of the monthly payment that reduces the outstanding balance.

State which of the following graphs best describes the changing value of I and B over the period of the loan, and give a valid reason for your answer.



(4)

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2.2 Nandi takes a home loan, to be amortised over 15 years, with interest charged at 11,25% per annum compounded monthly. Her first payment is made after eight months. After four years, the balance outstanding is R317 279,95.

(a) Calculate the monthly payment, correct to the nearest rand. (5)

(b) Calculate the amount of the original loan, correct to the nearest cent. (7)

(c) Find the total interest paid, in rand, over the first four years, that is, starting from when the loan was granted. (6)

(d) Nandi misses the 42^{nd} and 43^{rd} payments.

Given that the time period of the loan remains unchanged, calculate the revised payment required after the missed payments. (8)

[30]

QUESTION 3

A financial investment can be described by the following recurrence relation, where *F* is given in rand and *n* is the number of years.

$F_n = aF_{n-1} + b; \quad F_0 = 15\,000$

It is also given that $F_1 = 26\,200$ and $F_2 = 38\,296$

3.1 Interpret the meaning of *b* and F_0 in the context of the investment. (2)

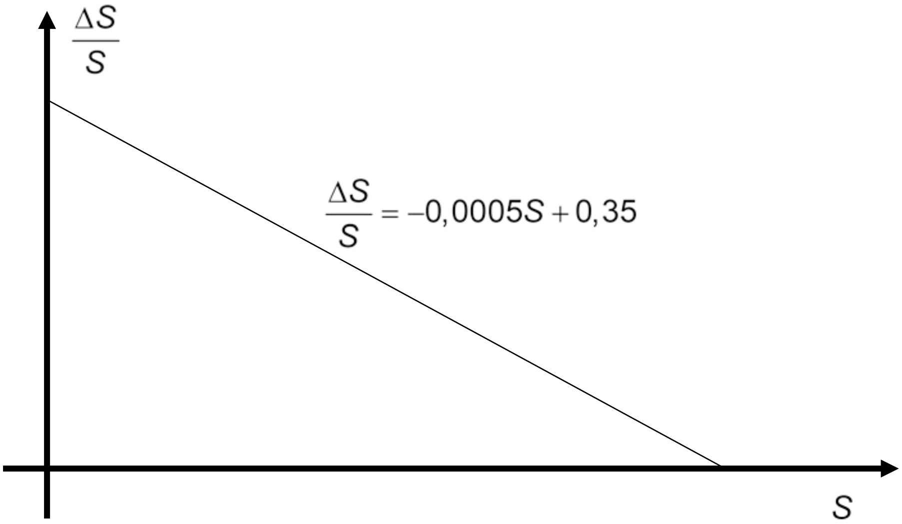
3.2 Calculate the values of *a* and *b* respectively. (6)

3.3 State the interest rate. (2)

[10]

QUESTION 4

Sandile rears sheep. He allows them to reproduce naturally, subject to the constraints of the farm environment. There are no predators but space and food resources would prevent them from continuing to increase in numbers exponentially. Sandile predicts that the sheep will increase in number according to the following model, illustrated graphically:



4.1 What type of population growth is Sandile illustrating? (1)

4.2 What is the maximum number of sheep expected? (3)

4.3 What is the intrinsic growth rate? (2)

4.4 Write the growth model as a recursive formula:

$S_{n+1} = \ldots$ where n is the number of years. (4)

4.5 If Sandile starts with 50 sheep, after how many years will the sheep population exceed half of the carrying capacity of the environment? (2)

[12]

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QUESTION 5

5.1 In a certain area of the African savanna, lions prey mainly on the zebra population. Since a ban on poaching is strictly applied, the lions have no predators themselves. The lion and zebra populations can be modelled effectively using the Lotka-Volterra recursive formulae:

$$L_{n+1} = L_n + f.bZ_nL_n - cL_n$$

$$Z_{n+1} = Z_n + aZ_n\left(1 - \frac{Z_n}{K}\right) - bZ_nL_n$$

The lions and zebras have now reached stable populations with approximately 1 000 zebras being hunted by eight lions.

Records suggest that a lion encountering a zebra results in a kill occurring in 5% of the cases.

The intrinsic growth rate of the zebra population is known to be 0,8 (80%) per annum.

(a) Explain the meaning of the term bZ_nL_n . (2)

(b) If the stable populations of lions and zebras are given by L_E and Z_E respectively, prove that:

$$L_E = \frac{a}{b}\left(1 - \frac{c}{Kfb}\right)$$ (10)

(c) Hence or otherwise, determine the carrying capacity, K. (5)

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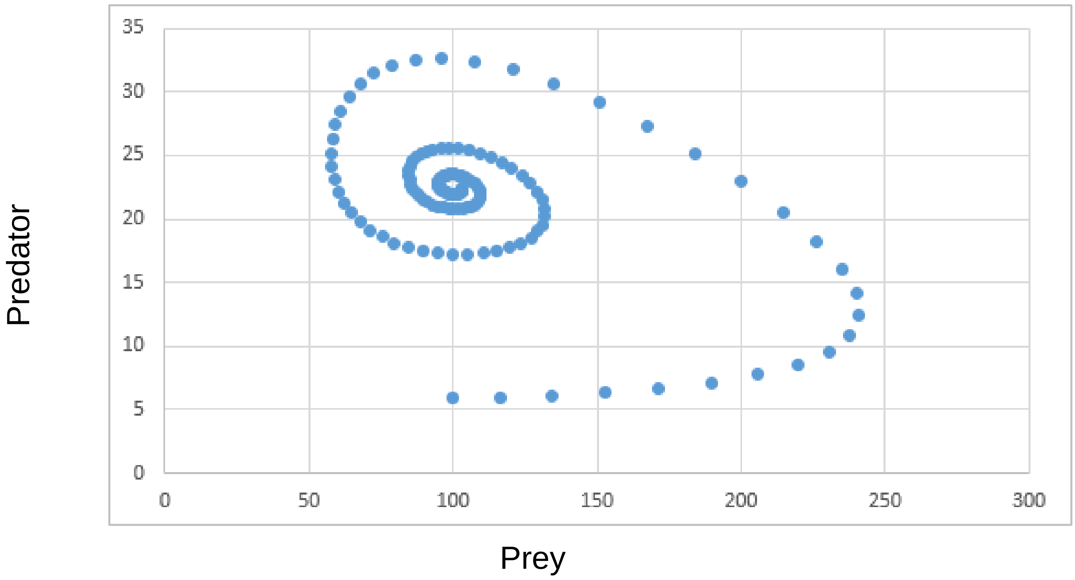
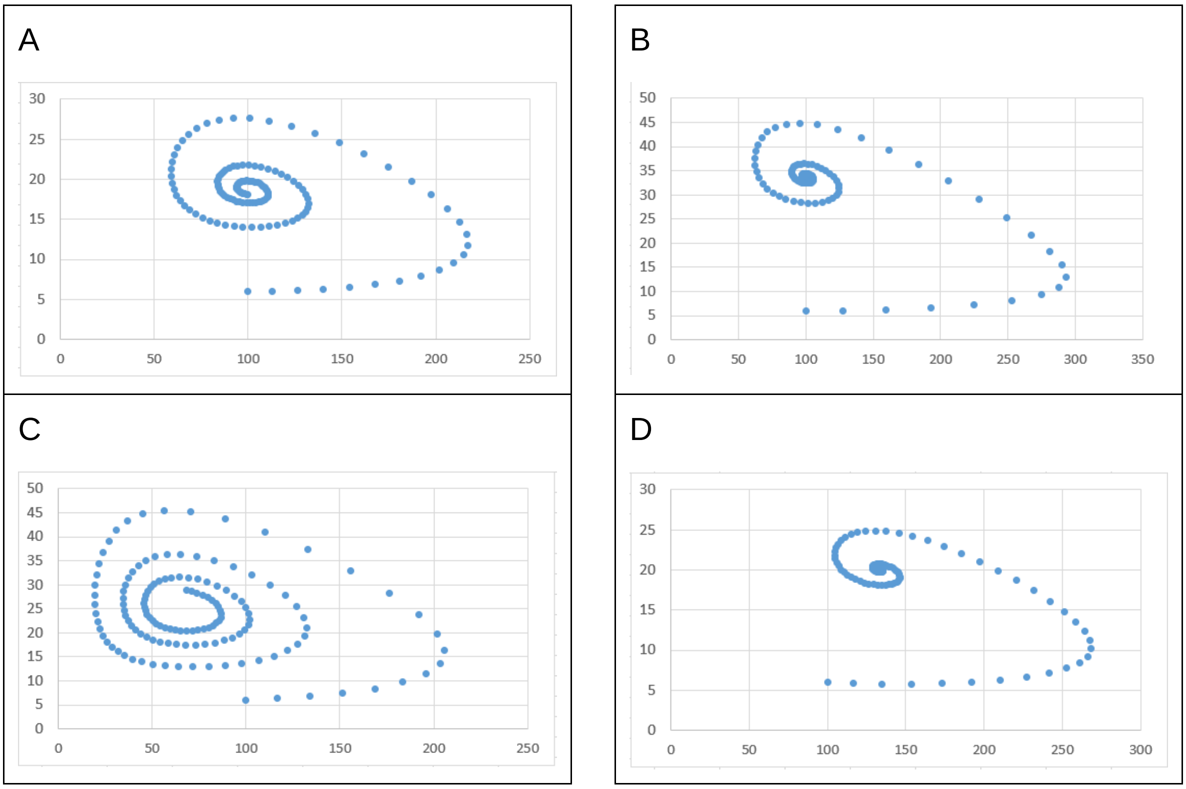
5.2 A phase plot of a predator-prey model is given below:



The following phase plots represent the same model with the value of the parameters adjusted.



State which of the graphs represents:

(a) an increase in the parameter '*a*' only. (2)

(b) an increase in the parameter '*f*' only. (2)
[21]

QUESTION 6

Consider a second-order difference equation of the form:

$u_n = au_{n-1} + bu_{n-2}$

The explicit formula for this sequence can be of the form:

$u_n = Ap^n + Bq^n$

where p and q are the roots of the quadratic equation:

$x^2 - ax + b = 0$

The constants A and B can then be solved using the initial values u_0 and u_1.

6.1 Determine the explicit formula related to the difference equation:

$u_n = 8u_{n-1} + 12u_{n-2}; \quad u_0 = -1; \quad u_1 = 6$ (7)

6.2 Determine the difference equation related to:

$u_n = 3 \times 2^n + 5 \times 3^n$ (5)

[12]

Total for Module 3: 100 marks

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MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 Given $\begin{pmatrix} 2 & 8 & 3 \\ 5 & 4 & 1 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ y & 3 \\ x & 4 \end{pmatrix} = \begin{pmatrix} 38 & 38 \\ 46 & 21 \end{pmatrix}$, solve for x and y. (7)

1.2 To solve the following equations simultaneously,

$x + 2y - z = -1$
$2x + 6y + z = 7$
$5x + 7y - 4z = 9$

we apply Gaussian row-reduction to the augmented matrix, $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 2 & 6 & 1 & 7 \\ 5 & 7 & -4 & 9 \end{array}\right)$

Step 1: $R_2 - 2R_1$ $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 2 & 3 & 9 \\ 5 & 7 & -4 & 9 \end{array}\right)$

Step 2: $R_3 - 5R_1$ $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 2 & 3 & 9 \\ 0 & -3 & 1 & 14 \end{array}\right)$

Step 3: $R_1 - R_2$ $\left(\begin{array}{ccc|c} 1 & 0 & -4 & -10 \\ 0 & 2 & 3 & 9 \\ 0 & -3 & 1 & 14 \end{array}\right)$

Continue the **Gaussian row reduction**, to solve for the variables x, y and z. (8)
[15]

QUESTION 2

2.1 Given, $P = \begin{bmatrix} 2 & 2 & 6 \\ 4 & 2 & 2 \end{bmatrix}$, calculate P', the image of P, using only matrix algebra,

(a) Translated one unit left and three units down. (3)

(b) Reflected in the $y = -x$. (4)

2.2 Find a simplified single matrix that first enlarges a shape by a factor of $\frac{2}{\sqrt{2}}$ and then rotates it 225°. (8)

2.3 A line with end points (3; 4) and (–1; 0) is reflected to points $\left(\frac{-3+4\sqrt{3}}{2};\frac{4+3\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2};\frac{-\sqrt{3}}{2}\right)$ respectively. Find the gradient of the line of reflection. (12)

[27]

QUESTION 3

3.1 State whether the given statements are true or false, giving a reason if false.

(a) In a given square matrix, if you multiply a row or column by a non-zero constant, the determinant is multiplied by that same non-zero constant. (2)

(b) If you interchange two columns in a determinant, the resulting determinant will be the same. (2)

(c) The determinant of a matrix will be zero if a row is a constant multiple of another row. (2)

3.2 Given A = $\begin{pmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 \\ 9 & 2 & 3 & 1 \end{pmatrix}$

Joe produced the following first step in calculating the determinant of a 4 × 4 matrix.

$$\begin{vmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 \\ 9 & 2 & 3 & 1 \end{vmatrix} = p\begin{vmatrix} 4 & 1 & 2 \\ 3 & 2 & 1 \\ 9 & 3 & 1 \end{vmatrix} + q\begin{vmatrix} 3 & 0 & 1 \\ 4 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix}$$

(a) Joe applied his knowledge of determinants to considerably simplify the process of finding the determinant. Describe what property of determinants Joe applied to produce his first step. (2)

(b) Determine the value of p and q. (2)

(c) Hence, using matrix algebra, show that the determinant is 24. (4)

[14]

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QUESTION 4

Select the most correct answer in each question:

4.1 What is the number of edges present in a complete graph with n vertices?

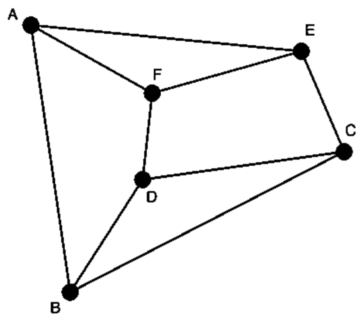
(a) $\frac{n(n+1)}{2}$

(b) $\frac{n(n-1)}{2}$

(c) n

(d) The information given is insufficient (2)

4.2 Is the following graph regular? Justify your answer.



(2)

4.3 The degree of each vertex of a connected graph is given below. Which graph or graphs are simple Eulerian circuits?

(a) 2, 4, 6, 4, 2

(b) 4, 1, 2, 3, 4

(c) 2, 2, 4, 2, 2

(d) 3, 3, 4, 3, 3 (2)

4.4 Which of the following graphs are isomorphic to each other?

Fig.1:

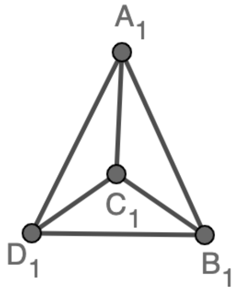


Fig.2:

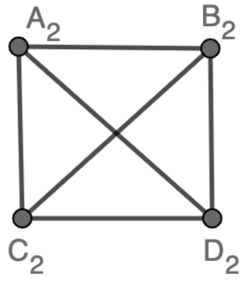
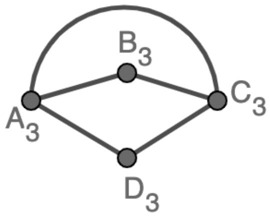


Fig.3:



(2)
[8]

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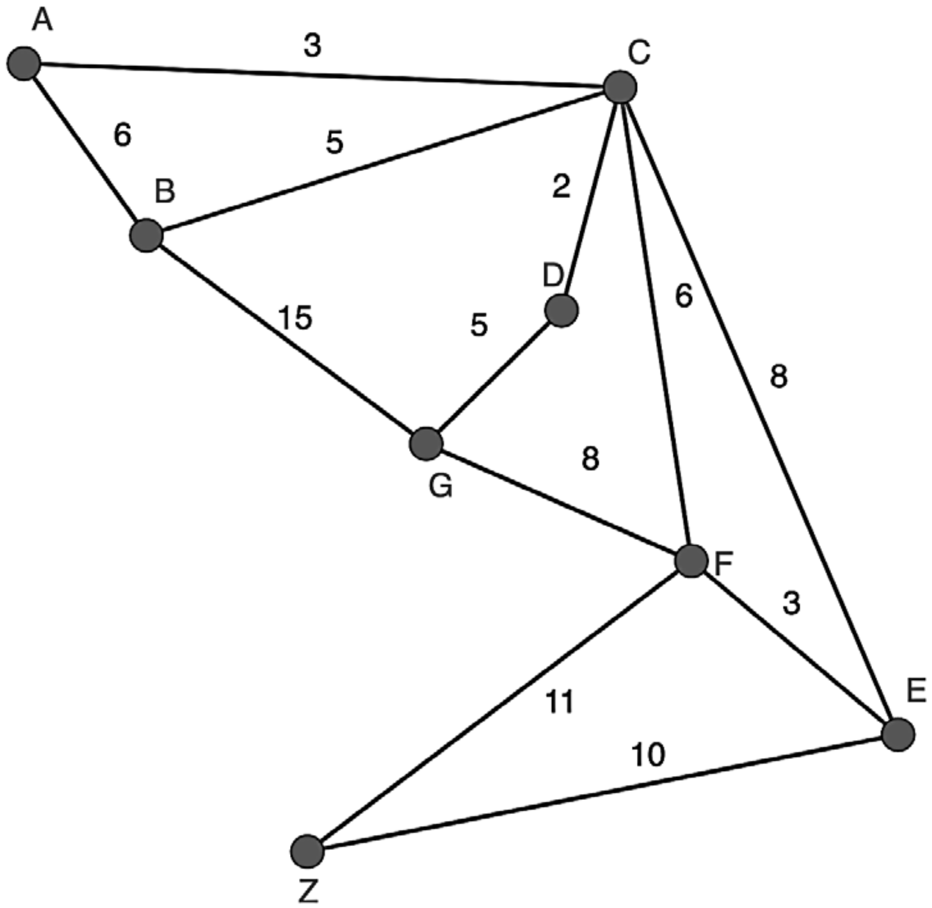
QUESTION 5

5.1 A Moova driver is parked at A and gets a request to collect friends from locations: B, C, D, E, F and G with instructions to drop them off at destination Z. The weights of the edges represent time in minutes.

Using an appropriate algorithm and removing vertex D, calculate the lower bound time of his journey.



(7)

5.2 A Moova driver has a separate request to travel from A to Z on the above graph (map). Help the driver by suggesting the **shortest route**. Show clear evidence of your working and reasoning. Answers only will get zero credit. (8)

5.3 The Moova driver thinks he has a short cut of time *t*, between vertices B and F. The driver is unsure of the exact time of this short cut but does know that, $2 < t \leq 4, t \in \mathbb{Z}$. Would you recommend he take the short cut? Justify your answer. (3)

[18]

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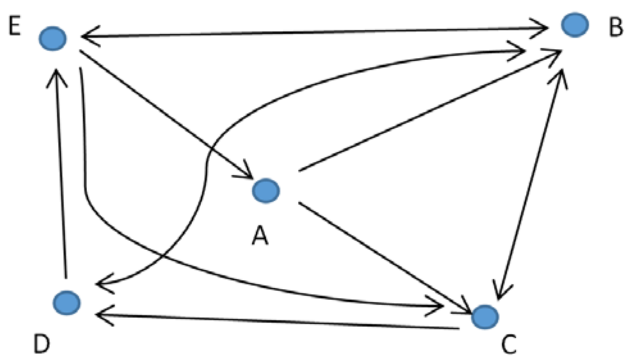
QUESTION 6

In a small town, some people have the cellphone numbers of others, but not everyone has every person's number. The directed graph below represents these relationships.

A ***directed graph*** has a specific direction to the value of the edge. In the graph below, it may be that Person A has Person B's cellphone number, but Person B does not have Person A's cellphone number in return, so the edge has an arrow (direction) from person A to B only, and not from B to A.



6.1 On the ANSWER SHEET complete the adjacency matrix, using the graph. Use a 1 to represent a directed edge.

From \ To	A	B	C	D	E
A		1	1	0	0
B					
C					
D					
E					

(4)

6.2 State the most connected person(s) in the graph. (2)

6.3 Create the directed complement of the graph. (4)

6.4 (a) Assuming that if you don't have the cellphone number of a person, there is an independent probability of 0,2 for each edge travelled that you can obtain the cellphone number. Weight the edges of your complement graph with the probabilities of getting the cellphone number of a new person. (6)

(b) Which two people were least likely to have each other's cellphone numbers? (2)

[18]

Total for Module 4: 100 marks

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